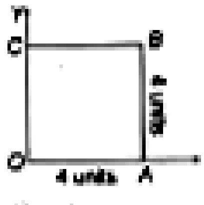
Grade X

Lesson : 7 Coordinate Geometry

Objective Type Questions

I. Multiple choice questions

1. In the given figure, O is the intersecting point of OA and OC and OABC is a square of side 4 units, then the position of A, B and C is



a) (4, 0) (4, 4) (0, 4)   b) (4, 0) (0, 4) (4, 4)

c) (0, 4) (4, 4) (4, 0)   d) None of the above

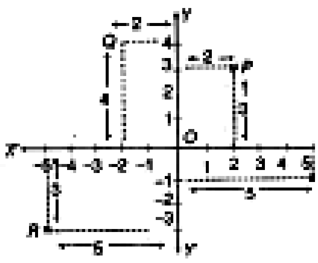
2. In the given figure, the ordinates of the points P, Q, R and S is

a) 2, -2, -5, 6

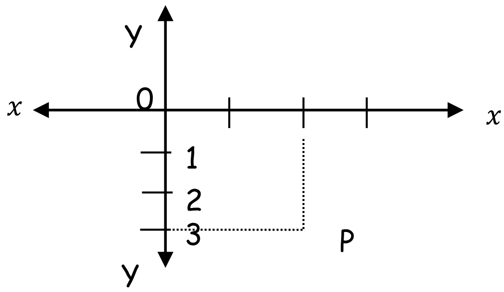
b) 3, 4 ,-3, -1

c) 3, 4, -5, 5

d) 2, 4, -5, -1



3. The coordinates of the point P as shown in the diagram will be



a) (2, -3)   b) (-3, 2)   c) (2, 3)   d) (3, 2)

4. The coordinate of the vertices of a rectangle whose length and breadth are 6 and 4 units, respectively. Its one vertex is at the origin. The longer side is on the x – axis and one of the vertices lies in second quadrant is

a) (0, 0) (6, 4) (6, 0) (0, 4)   b) (0, 0) (0, 4) (6, 0) (6, 4)

c) (0, 0) (6, 4) (-6, 0) (6, 4)   d) (0, 0) (6, 4) (-6, 4) (-6, 0)

5. The point P (-4, 2) lies on the line segment joining he points A (-4, 6) and B (-4, -6)

a) True   b) False

c) Can't say   d) Partially True / False

6. The distance of the point P (2, 3) from the x axis is

a) 2 units   b) 3 units   c) 1 units   d) 5 units

7. The distance between the points P (-6, 7) and Q (-1, -5) is

a) – 6 units   b) 13 units   c) 1 units   d) 5 units

8. The distance between the points (a cos θ + b sin θ, 0) and

(0, a sin $\theta -$ b cos θ), is

a) $a^2 + b^2$ b) $a^2 - b^2$ c) $\sqrt{a^2 + b^2}$ d) $\sqrt{a^2 - b^2}$

9. The value of y, if the distance between the points (2, y) and (-4, 3) is 10 is

a) 6   b) -11   c) 5   d) 11

10. Point P (0, 2) is the point of intersection of y - axis and perpendicular bisector of line segment joining the points A(1, 1) and B (3, 3)

a) True   b) False

c) Can't say   d) Partially True / False

11. If the distance between the points (4, p) and (1, 0) is 5, then the value of p is.

a) 4   b) -4   c) Both a and b   d) 0

12. A circle has its centre at the origin and a point P (5, 0) lies on it. The point Q (6, 8) lies outside the circle.

a) True   b) False

c) Can't say   d) Partially True / False

13. The radius of the circles whose centre is at (0, 0) and which passes through the points (-6, 8) is _______.

a) 10 units   b) 11 units   c) 9 units   d) 8 units

14. Is the points (1 , -1), (5, 2) and (9, 5) are collinear?

a) Yes b) No

c) Can't find d) None of the above

15. If the point P (2, 1) lies on the line segment joining points A(4, 2) and B (8, 4), then ________.

a) AP = $\frac{1}{3}$ AB b) AP = PB c) PB = $\frac{1}{3}$ AB d) AP = $\frac{1}{2}$ AB

16. If the point P (x, y) is equidistant from the points A (5, 1) and B (1, 5), then The points A(5, 1)and B (1, 5) then

a) y = 3x b) x = y c) x = -8y d) -8x = y

17. A point on x - axis which is equidistant from the points (1, 3) and (-1, 2)

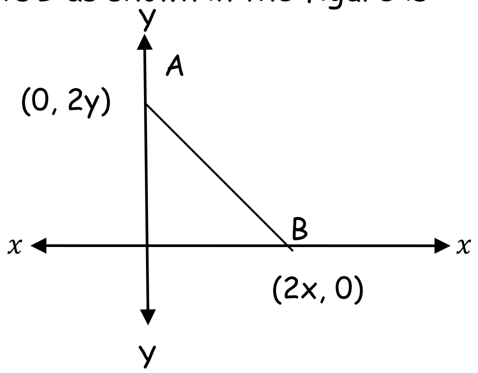
a) $\left(\frac{5}{2}, 0\right)$ b) (5, 0) c) (4, 0) d) $\left(\frac{5}{4}, 0\right)$

18. The point on x- axis which is equidistant from the point (7, 6) and (-3, 4) is

a) (0, 3) b) (4, 3)

c) (3, 0) d. None of these

19. The coordinates of the point which is equidistant from the three vertices of the Δ AOB as shown in the figure is



a)(x, y) b) (y, x) c) $\left(\frac{y}{2}, \frac{x}{2}\right)$ d) $\left(\frac{x}{2}, \frac{y}{2}\right)$

20. The coordinate of a point on y – axis which is equidistant from the point A (6, 5) and B (-4, 3) will be

a) (0, 9) b) (0, -9) c) (0, 5) d) (0, 3)

21. The radius of the circle whose and points of diameter are (24, 1) and (2, 23) is

a) $22\sqrt{2}$ units　　b) $23\sqrt{2}$ units

c) $11\sqrt{2}$ units　　d) None of these

22. If the points A (4, 3) and B(x, 5) are on the circle with centre O (2, 3), then the value of x is

a) 0　　b) 1　　c) 2　　d) 3

23. The perimeter of a triangle with vertices (0, 4) (0, 0) and (3, 0) is

a) 5 units　　b) 12 units　　c) 11 units　　d) (7+ $\sqrt{5}$ units)

24. If three points (0, 0), (3, $\sqrt{3}$) and (3, λ) from an equilateral triangle then - λ equals.

a) 2　　b) -3

c) -4　　d) None of these

25. If AOBC is a rectangle whose three vertices are A (0, 3), O(0, 0) and B (5, 0), then the length of its diagonal is

a) 5 units　　b) 3 units　　c) $\sqrt{34}$ units　　d) 4 units

26. The points (3, 2), (-2, -3) and (2, 3) form a triangle name the type of triangle formed.

a) equilateral　　b) isosceles

c) right angle　　d) None of these

27. (5, -2) (6, 4) and (7, -2) are the vertices of an ________triangle.

a) equilateral　　b) right angle

c) isosceles　　d) None of these

28. The points (-4, 0), (4, 0) and (0, 3) are the vertices of a

a) right angled triangle　　b) isosceles triangle

c) isosceles　　d) None of these

29. The points (2, 3), (3, 4), (5, 6) and (4, 5) are the vertices of a

a) Parallelogram　　b) Triangle

c) Square　　d) None of these

30. The coordinates of the point which divides the line segment joining the points (4, -3) and (9, 7) internally in the ration 3 : 2 is.

a) (7, 3) b) (3, 7) c) (35, 15) d) (27, 21)

31. The point which divides the line segment joining the points (7, -6) and (3, 4) in the ratio 1 : 2 internally lies on the

a) i quadrant b) ii quadrant c. iii quadrant d) iv quadrant

32. If P (9a – 2 - b) divides line segment joining A (3a + 1, -3) and B (8a, 5) in the ratio 3 : 1, then the values of a and b is

a) a = -1, b = 3 b) a = -1, b = -3 c) a = 0, b = 0 d) a = 1, b = -3

33. The point (-4, 6) divides the line segment joining the points A (-6, 10) and B (3, -8) The points A (-6, 10) and B (3, 8) The ratio is

a) 1 : 2 b) 7 : 2 c) 2 : 7 d) 4 : 1

34. If P $\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q (-6, 5) and R (-2, 3) then the value of a is.

a) -4 b) -12 c) 12 d) -8

35. The fourth vertex D of a parallelogram ABCD whose three vertices are A (-2, 3), B (6, 7) and C (8, 3) is

a) (0, 1) b) (0, -1) c) (-1, 0) d) (1, 0)

36. If x – 2y + k = 0 is a median of the triangle whose vertices are at points A = (-1, 3), B (0, 4) and C (-5, 2), then the value of k is

a) 2 b) 4 c) 6 d) 8

37. The perpendicular bisector of the line segment joining the points A(1, 5) and B (4, 6) cuts the y – axis at

a) (0, 13) b) (0, -13) c) (0, 12) d) (13, 0)

38. The point _____ lies on the perpendicular bisector of the line segment joining the points A(-2, -5) and B (2, 5)

a) (0, 0) b) (0, 2) c) (2, 0) d) (-2, 0)

39. A line intersects the y axis and x axis at the points P and Q respectively. If (2, -5) is the midpoint of PQ then the coordinated of P and Q are respectively.

a) (0, 5) and (2, 0) b) (0, 10) and (-4, 0)

c) (0, 4) and (-10, 0) d) (0, -10) and (4, 0)

40. Δ ABC with vertices A (-2, 0) B (-2, 0) and C (0, 2) is similar to Δ DEF with vertices D (-4, 0), E (4, 0) and F (0, 4)

a) True b) False

c) Can't say d) Partially True / False

41. If (a, b) is the mid-point of the line segment joining the points A(10, -6) and B (k, 4) and a - 2b = 1, then the value of k is

a) 30 b) 22 c) 4 d) 40

42. Using section formula, check that the points A(-3, -1), B (1, 3) and C(-1, 1) are collinear

a) Yes b) No

c) Can't say d) None of these

43. The ratio, in which the Y axis divides the line segment joining the point (5, -6) and (-1, -4) is

a) 1 : 5 b) 5 : 1

c) 2 : 4 d) None of these

44. Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2:3

a) (1, 3) b) (2, 6) c) (3, 4) d) (4 , 6)

45. The ratio in which the point P (m, 6) divides the join A (-4, 3) and B (2, 8) is ___________.

a) 2: 3 b) 1: 2 c) 3: 2 d) 2: 1

46. If the points A (6, 1), B (8, 2) C (9, 4) and D (p, 3) are the vertices of a parallelogram, taken in order, then the value of p is.

a) 5 b) 6 c) 8 d) 7

47. The coordinates of point A, where AB is the diameter of a circle whose centre is (3, -4) and B is (1, 4) is '

a) (2, 0) b) (12, -5)

c) 5, -12) d) None of the above

48. The coordinates of the point of trisection of the line segment joining (2, -3) and (4, -1) (when the point is near the point (2, -3) is

a) (10/3, -5/3) b) (8/3, - 7/3)

c) (3, -2) d) None of the above

49. List II gives the coordinates of the point P that divides the line segment joining the points in the given ratio given in List-I, match them correctly.

List I		**List II**	
P.	A (-1, 3) and B (-5, 6) internally in the ratio 1 : 2	1.	(7, 3)
Q.	A(-2, 1) and B (1 ,4) internally in the ratio 2: 1	2.	(0, 3)
R.	A(1, 7) and B (1, 4) internally in the ratio 2: 1	3.	$\left(\frac{11}{3}, \frac{26}{3}\right)$
S.	A(4, -3) and B (8, 5) internally in the ratio 3: 1	4.	$\left(\frac{7}{3}, 4\right)$

Codes

	P Q R S		P Q R S
a)	4 2 3 1	b)	3 2 4 1
c)	4 2 3 1	d)	3 1 2 4

50. Match the following

	List I		List II
P.	Distance between (-6, 7) and (-1, -5) is	1.	-3, 7
Q.	The value of k for which the distance between A (k, -5) and B (2, 7) is 13 units	2.	x + y = 5
R.	(x, y) is equidistant from (5, 1) and (-1, 5) if	3.	$3x = 2y$
S.	x, y (2, 3) and (8, 5) internally in the ratio 3: 1	4.	$\left(\frac{7}{3}, 4\right)$

P Q R S	P Q R S	P Q R S	P Q R S
a) 3 4 2 1	b) 1 2 4 3	c) 4 1 3 2	d) 3 2 4 1

II. Multiple choice questions

1. The ratio of the distances of point P (3, 4) from origin to that from y – axis is.

   a) 3 : 5 b) 5 : 3 **c) 5 : 4** d) 3 : 4

2. AOBC is a rectangle whose three vertices are A(0, 4) , O (0, 0) and B (3, 0). The length of its diagonal is.

   a) 25 units **b) 5 units** c) 3 units d) 4 units

3. The distance between A (10 cos θ, 0) and B (10 sin θ, 0) is.

   a) $\sqrt{10}$ **b) 10**

   c) 5 d) 10 sin θ, cos θ

4. Perimeter of the triangle formed by the points O (0, 0), A(a, 0) and B (0, b) is

   a) a + b b) ab

   c) a + b +2$\sqrt{ab}$ **d) a + b +** $\boldsymbol{\sqrt{a^2 - b^2}}$

5. The points (-8, 0) , (8, 0), (0, 5) are the vertices of a

   a) right triangle **b) isosceles triangle**

   c) equilateral triangle d) scalene triangle

6. The points (-2, 2) (8, -2) and (-4, -3) are the verticies of a

a) equilateral triangle
b) isosceles triangle
c) right triangle
d) scalene triangle

7. The points (1, 7) (4, 2) (8, -2) and (-4, -3) are the verticies of a

a) parallelogram
b) rhombus
c) rectangle
d) square

8. The line segment joining the points (2, -3) and (5, 6) is divided by x- axis in the ratio.

a) 2 : 1 b) 3 : 1 c) 1: 2 d) 1 : 3

9. The line segment joining the points (3, 5) and (-4, 2) is divided by y-axis in the ratio.

a) 5 : 3 b) 3 : 5 c) 4 : 3 **d) 3 : 4**

10. If (3, 2), (4, k) and (5, 3) are collinear then k is equal to

a) $\frac{3}{2}$ b) $\frac{2}{5}$ **c)** $\frac{5}{2}$ d) $\frac{3}{5}$

11. If the points (p, 0), (0, q) and (1, 1) are collinear then $\frac{1}{p}+\frac{1}{q}$

a) -1 **b) 1** c) 2 d) 0

12. The coordinates of reflection of Q (-1, -3) in x-axis are

a) (1, 3)
b) (-1, 3)
c) (1, -3)
d) none of these

13. The distance between the points (-3, 0) and (3, 0) is

a) 3 units b) $3\sqrt{2}$ units c) $2\sqrt{3}$ units **d) 6 units**

14. If P $\left(\frac{a}{3},\ 4\right)$ is the mid point of the line segment joining the points A (-6, 5) and B (-2, 3) then value of a is.

a) -4 **b) -12** c) 12 d) -6

15. The fourth vertex D of a parallelogram ABCD whose three vertices are A (- 2, 3), B (6, 7) and C (8, 3) is

a) (0, 1) **b) (0, -1)** c) (1, 0) d) (-1, 0)

Fill in the blanks

1. Distance of point P (a, b) from origin is ___________

   $\sqrt{a^2 + b^2}$

2. Coordinates of mid point joining P (α, β) and Q (γ, δ) are ___________

   $\left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right)$

3. For given three points A,B, C if out of three possible distances A, B, C if out of three possible distance AB, BC and CA the length of the greatest distance is equal to sum of other two distances than the points A, B , C are said to be **Collinear**

4. For given four points A, B, C, D if lengths, AB, BC, CD and DA are all equal then ABCD is necessarily a **rhombus**

5. For given four points A, B, C, D if length AB = BC = CD = DA and AC ≠ BD then ABCD is a **rhombus** but not **square**

6. For given four points ABCD to be a **Parallelogram** It is sufficient to show that opposite sides are equal.

7. For given four points A, B, C, D if AB = CD BC = DA and AC ≠ BD then ABCD is a **Parallelogram** but not **rectangle**.

8. If area of a triangle is zero square units then its verticies are **collinear**

9. The distance between (α, β) and $(-\alpha - \beta)$ is $2\sqrt{\alpha^2 + \beta^2}$.

10. If the point P (x, y) divides the line segment joining A (x_1, y_1) and B (x_2, y_2) In the ratio m : n the value of y - x = $\frac{m(y_2 - x_2) + n(y_1 - x_1)}{m+n}$

I. Very short answer question

1. A triangle with verticies (4, 0), (-1, -1) and (3, 5) is a / an

a) equilateral triangle
b) right- angled triangle
c) isosceles right – angled triangle
d) none of these

Let A (4, 0), B (-1, -1), C (3, 5)

AB = $\sqrt{(-1-4)^2 + (-1-0)^2}$ = $\sqrt{26}$

BC = $\sqrt{(3+1)^2 + (5+1)^2}$ = $\sqrt{52}$

AC = $\sqrt{(3-4)^2 + (5-0)^2}$ = $\sqrt{26}$

$\Rightarrow$ AB^2 + AC^2 - BC^2 and AB = AC

Hence, triangle is an isosceles right angled triangle

2. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

a) $\left(\frac{3}{4}, 1\right)$ b) $\left(2, \frac{7}{3}\right)$ c) $\left(5, \frac{1}{2}\right)$ **d) $\left(-6, \frac{5}{2}\right)$**

Distance of $\left(-6, \frac{5}{2}\right)$ from centre of the circle i.e. (0, 0)

= $\sqrt{(0+6)^2 + \left(0-\frac{5}{2}\right)^2}$ = $\sqrt{36 + \frac{25}{4}}$

= $\sqrt{\frac{144+25}{4}}$ = $\frac{13}{2}$ = radius circle

3. If the distance between the points (4, p) and (1, 0) is 5 units, then the value of p is

a) 4 only **b) $\pm$ 4** c) -4 only d) 0

b) $\sqrt{(4-1)^2 + (p-0)^2}$ = 5

$\Rightarrow$ $3^2 + p^2 = 5^2 \Rightarrow p^2$ = 25 - 9 = 16 $\Rightarrow$ p = ± 4

4. Find the distance of a point P (x, y) from the origin

Let the coordinates of the origin be O (0, 0)

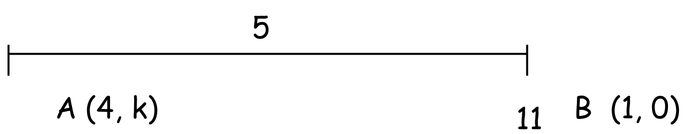
By using distance formula

Distance PO = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance PO = $\sqrt{(0-x)^2 + (0-y)^2}$

= $\sqrt{(-x)^2 + (-y)^2}$ = $\sqrt{(x)^2 + (y)^2}$ units

5. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k?



Using distance formula

AB = $\sqrt{(1-4)^2 + (0-k)^2}$

⇒ 5 = $\sqrt{(-3)^2 + (-k)^2}$

⇒ 5 = $\sqrt{9 + k^2}$

On squaring, we get

$(5)^2 = \left(\sqrt{9+k^2}^{\,2}\right) = 9 + k^2$

⇒ 25 - 9 = k^2 ⇒ k^2 = 16

∴ k = $\pm\sqrt{16} = \pm 4$

6. Show that (1,-1) is the centre of the circle circumscribing the triangle whose angular points are (4, 3), (-2, 3) and (6, -1)

P (1, -1) will be the centre of the circle circumscribing the triangle whose angular points are A (4, 3) B (-2, 3) and C (6, -1) if PA = PB = PC

Now PA= $\sqrt{(4-1)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25}$ = 5

PB = $\sqrt{(-2-1)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25}$ = 5

PC = $\sqrt{(-2-1)^2 + (3+1)^2} = \sqrt{25}$ = 5

Hence the result

7. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, -3) and B is the point (1, 4)

AB is diameter of the circle.

Let C be centre of circle, coordinates of C are (2, -3)

So C is mid-point of AB (diameter)

Let coordinates of A are (x, y)

∴ $\frac{x+1}{2}$ = 2 and $\frac{y+4}{2}$ = -3

⇒ x + 1 = 4 and y + 4 = -6

⇒ x = 3 and y = -10

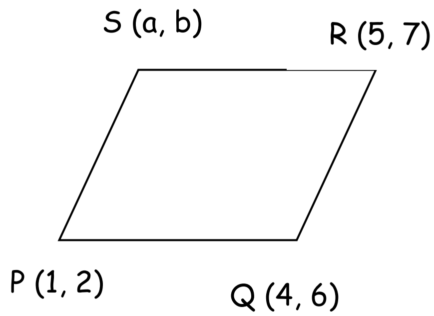
∴ the coordinates of A are (3, -10)

8. If P(1, 2) Q (4 ,6), R (5, 7) and S (a, b) are the vertices of a parallelogram PQRS, then

a) a = 2, b = 4 b) a = 3, b = 4 c) a = 2, b = 3 d) a = 3 b = 5

Mid – point of PR = $\left(\frac{1+5}{2}, \frac{2+7}{2}\right)$ = $(3, \frac{9}{2})$

Mid – point of SQ = $\left(\frac{4+a}{2}, \frac{6+b}{2}\right)$



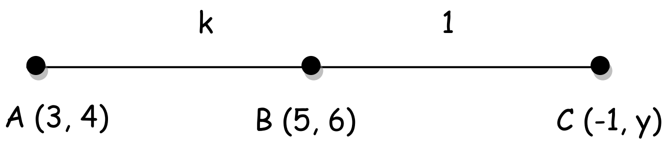
Diagonal of parallelogram bisect each other.

∴ $(3, \frac{9}{2}) = \left(\frac{4+a}{2}, \frac{6+b}{2}\right)$

⇒ $3 = \frac{4+a}{2}, \frac{9}{2} = \frac{6+b}{2}$

⇒ a = 2 b = 3

9. A straight line is drawn joining the points (3, 4) and (5, 6). If the line is extended, the ordinate of the point on the line, whose abscissa is -1 is _____.



Let line is extended, C (-1, y) such that

AB = BC = K : 1

∴ $\frac{(-1) \times K + 3}{K + 1} = 5$

⇒ $K = -\frac{1}{3}$

∴ Y = 0

10. A (5, 1), B (1, 5) and C (-3, -1) are the vertices of Δ ABC. Find the length of median AD.

AD is the median of Δ ABC

∴ D is mid - point of BC

Coordinates of D are :

$\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = \left(\frac{-2}{2}, \frac{4}{2}\right)$ = (-1, 2)

Length of AD = $\sqrt{(5+1)^2(1-2)^2}$

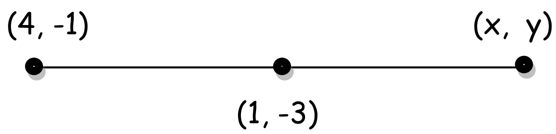
$= \sqrt{36+1} = \sqrt{37}$ units

11. Find the coordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0)

Coordinates of the centroid of a triangle whose vertices are (x_1, y_1) (x_2, y_2) (x_3, y_3) are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + + y_3}{3}\right)$

$= \left(\frac{0 + 8 + 8}{3}, \frac{6 + 12 + 0}{3}\right) = \left(\frac{16}{3}, \frac{18}{3}\right) = \left(\frac{16}{3}, 6\right)$

12. The coordinates of one end point of the diameter is (4, -1) and centre of the circle is (1, -3). Find the coordinates of the other end of the diameter.



Given that coordinates of one end point of the diameter is (4, -1) and centre of the circle is (1, -3)

Let coordinates of the other end of the diameter be $(x$, y)

We know that the centre of the circle (1, -3) is the mid-point of diameter.

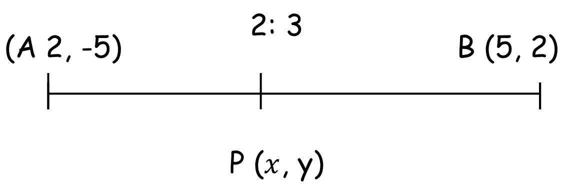
$\Rightarrow \quad \frac{4 + x}{2} = 1$ and $\frac{-1 + y}{2} = -3$

$\Rightarrow \quad 4 + x = 2$ and $-1 + y = -6$

$\Rightarrow \quad x = -2$ and $y = -6 + 1 = -5$

Thus, coordinates of the other end of the diameter are (-2, -5)

13. Point P divides the line segment joining the points A (2, -5) and B (5, -2) in the ratio 2: 3. Name the quadrant in which P lies.



$$x = \frac{2 \times 5 + 3 \times 2}{2 + 3}$$

$$y = \frac{2 \times 2 + 3(-5)}{2 + 3}$$

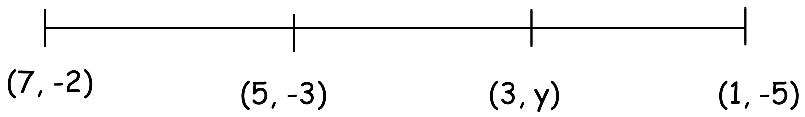
$\Rightarrow \quad x = \frac{10 + 6}{5} = \frac{16}{5} = 3.2$

$\Rightarrow \quad y = \frac{4 - 15}{5} = \frac{-11}{5} = -2.2$

Point P (3.2, - 2.2) lies in IV quadrant

14. In figure P (5, -3) and Q (3, y) are the points of trisection of the line segment joining A (7, -2) and B (1, -5) Find y

$\because$ AP = PQ = BQ



$\Rightarrow$ Q is mid-point of PB

$\Rightarrow \quad y = \frac{-3 + (-5)}{2} = -4$

15. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k?

Let A (4, k), B (1, 0) $\Rightarrow$

AB = 5 given

$\Rightarrow \quad \sqrt{(4-1)^2 + (k-0)^2} = 5$

$\Rightarrow \quad \sqrt{3^2 + k^2} = 5$

$\Rightarrow \quad k^2 + 9 = 25$

$\Rightarrow \quad k^2 = 25 - 9 = 16$

$\therefore \quad k = \pm 4$

16. Find the distance of a point P(x, y) from the origin. Using distance formula for distance between

A (x_1, y_1) and B (x_2, y_2)

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ Reqd distance = $\sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$

17. Find distance between A$(10\ \cos\theta, 0)$ and B $(0, 10\ \sin\theta)$

AB $= \sqrt{(0 - 10\ \cos\theta)^2 + (10\ \sin\theta - 0)^2}$

$= \sqrt{100\ cos^2 0 + 100\ sin^2\theta}$

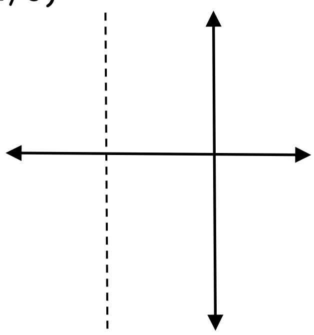
$= \sqrt{100\ (cos^2\theta + \ sin^2\theta\ )}$

$= \sqrt{100} = 10$ units

18. Find the coordinates of reflection of Q (-1, -3) in x-axis.

Reflection of Q (-1, -3) is (-1, 3)

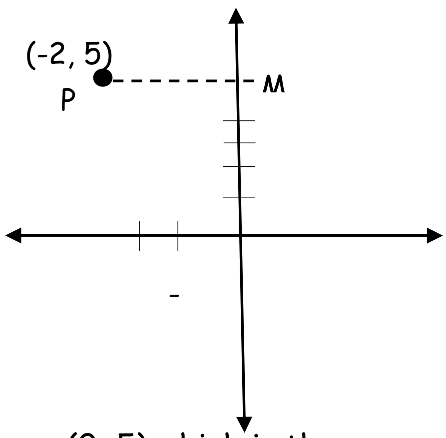
Q (-1, -3)



Q (-1, -3)

19. Find the coordinates of the point on y-axis which is nearest to the point (-2, 5)

To get the coordinates of the point on y - axis which is nearest to the point P (-2, 5) drop a perpendicular PM from on y-axis



Coordinates of M are (0, 5) which is the nearest point to P (-2, 5)

20. If the point (0,2) is equidistant from the points (3,k) and (k,5), find the value of k,

Let the points be P(0, 2), A(3, k) and B(k, 5)

Now, PA = PB

Or, $PA^2 = PB^2$

$\Rightarrow$ $(3-0)^2 + (K-2)^2 = (K-0)^2 + (5-2)^2$

$\Rightarrow$ $9 + (K-2)^2 = k^2 + 9$

$\Rightarrow$ $k^2 = (k-2)^2$

$\Rightarrow$ k $= \pm(k-2)$

k = (k - 2) (impossible)

$\therefore$ k = - (k - 2) = - k + 2

Or 2k = 2 or k = 1.

Short answer type questions I

1. Find the linear relation between x and y such that P (x, y) is equidistant from the points A (1, 4) and B (-1, 2)

$P(x, y)$ is equidistant from the points A(1, 4) and B(-1, 2)

PA = PB

$\sqrt{(x-1)^2 + (y-4)^2} = \sqrt{(x-1)^2 + (y-2)^2}$

Squaring both sides , we get

$(x-1)^2 + (y-4)^2 = (x-1)^2 + (y-2)^2$

$x^2 - 2x + 1 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2 - 4y + 4$

$-2x + 17 - 8y = 2x - 4y + 5$

$-4x - 4y = -12$

$x + y = 3$

2. If the point (x, y) is equidistant from the points (a + b, b - a) and (a - b, a + b), Prove that bx = ay

Consider that point P (x, y) is equidistant from

A(a + b, b - a) and B (a - b, a + b)

∴ PA = PB

$\sqrt{[x-(a+b)]^2+[y-\ (b-a)]^2}$

= $\sqrt{[x-(a-b)]^2+[y-\ (a+b)]^2}$

Squaring both sides , we get

x^2 + $(a+b)^2$ - 2(a +b) x + y^2 + $(b-a)^2$ - 2(b - a) y

= x^2 + $(a-b)^2$ - 2(a - b) x + y^2 + $(a+b)^2$ – 2y(a + b)

⇒ -2(a + b) x - 2(b - a) y = -2(a - b)x -2y (a + b)

⇒ 2x(a - b) + 2y(a + b) y = 2x(a + b) +2y (b - a)

⇒ x(a - b) + y (a + b) = x(a + b) + y (b - a)

⇒ x(a – b - a - b) = y (b – a - a - b)

-2bx = -2ay

bx = ay

3. Find the point on y-axis which is equidistant from the points (5, -2) and (-3, 2)

Let point on y-axis be (0, a)

Now distance of this point from (5, -2)is equal to distance from point (-3,2)

i.e. $\sqrt{5^2+(-2-a)^2} = \sqrt{(3)^2+(a-2)^2}$

Squaring and simplifying we get

25 + 4 + a^2+ 4a = 9 + a^2+ 4- 4a

⇒ 8a = -16 ⇒ a = -2

Point (0, -2)

4. Write the coordinates of a point P on x - axis which is equidistant from the points A (-2, 0) and B(6, 0)

Let coordinates of the P are (x, 0)

ATO

AP = BP

$\sqrt{(x+2)^2+(0-0)^2} = \sqrt{(x-6)^2+(0-0)^2}$

$(x+2)^2$ = $(x-6)^2$

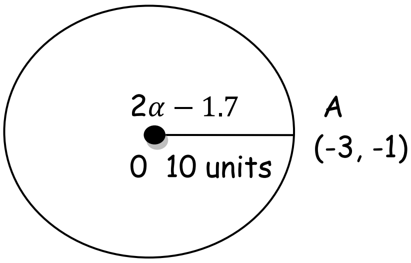
x^2 + 4 + 4x = x^2 + 36 -12x

16x = 32 ⇒ x = 2

∴ Coordinates of the point P are (2, 0)

5. The centre of a circle is (2α -1, 7) and it passes through the point (-3, -1). If the diameter of the circle is 20 units then find the value of α.

OA = 10 units.



⇒ OA = $\sqrt{(2\alpha - 1 + 3)^2 + (7+1)^2}$

⇒ 10 = $\sqrt{4\alpha^2 + 4 + 8\alpha + 64}$

Squaring 100 - $4\alpha^2 + 8\alpha$ + 68

⇒ $4\alpha^2 + 8\alpha$ -32 = 0 ⇒ $\alpha^2 + 2\alpha$ – 8 = 0

⇒ $\alpha^2 + 4\alpha$ -2α -8= 0 ⇒ α $(\alpha + 4)$ -2 $(\alpha + 4)$ = 0

⇒ $(\alpha + 4)$ $(\alpha - 2)$ = 0

∴ $\alpha = -4$ $\alpha = 2$

6. Use distance formula to show that the points A(-2, 3), B (7, 0) are collinear

AB = $\sqrt{(1+2)^2 + (2-3)^2}$ = $\sqrt{9+1}$ = $\sqrt{10}$q

BC = $\sqrt{(7-1)^2 + (0-2)^2}$ =$\sqrt{36+4}$ = $\sqrt{40}$ = 2$\sqrt{10}$

AC = $\sqrt{(7+2)^2 + (0-3)^2}$

= $\sqrt{81+9}$ = $\sqrt{90}$ = 3$\sqrt{10}$

Since AB + BC = $\sqrt{10}$ + 2$\sqrt{10}$

= (1 + 2) $\sqrt{10}$ = 3$\sqrt{10}$ = AC

Hence the points A, B and C are collinear.

7. Show that the points A (a, a), B (-a, -a) and C (-a$\sqrt{3}$, $a\sqrt{3}$) form an equilateral triangle.

AB $= \sqrt{(-a-a)^2 + (-a-a)^2}$

$= \sqrt{(-2a)^2 + (-2a)^2}$

$= \sqrt{4a^2 + 4a^2} = \sqrt{8a^2} = 2\sqrt{2}a$

BC $= \sqrt{\left(-a\sqrt{3} + a\right)^2 + \left(a\sqrt{3} + a\right)^2}$

$= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + a^2 + 2\sqrt{3}a^2}$

$= \sqrt{8a^2} = 2\sqrt{2}a$

AC $= \sqrt{\left(-a\sqrt{3} - a\right)^2 + \left(a\sqrt{3} - a\right)^2}$

$= \sqrt{3a^2 + a^2 + 2\sqrt{3}a^2 + 3a^2 + a^2 - 2\sqrt{3}a^2}$

$= \sqrt{8a^2} = 2\sqrt{2}a$

Since AB = BC = AC ∴ Δ ABC is equilateral triangle

8. Find the ratio in which P (4, m) divides the line segment joining the points A(2, 3) and B (6, -3) Hence find m.

Let p (4, m) divides A (2, 3) and B (6, -3) in the ration $m_1 : m_2$,

[By using section formula]

$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$, y $= \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$,

⇒ $4 = \frac{m_1 \text{ x } 6 + m_2 \text{ x } 2}{m_1 + m_2}$

⇒ $4m_1 + 4m_2 = 6m_1 + 2m_2$

⇒ $4m_2 + 2m_2 = 6m_1 + 2m_1$

⇒ $2m_1 + 2m_1$

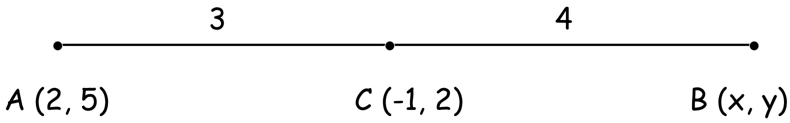
∴ $m_1 : m_2 = 1 : 1$

Also, m $= \frac{m_1 \text{ x } (-3) + m_2 \text{ x } 3}{m_1 + m_2}$

$$= \frac{1 \times (-3) + 1 \times 3}{2} = \frac{-3+3}{2} = \frac{0}{2} = 0$$

$\therefore$ m = 0

9. If the point C (-1, 2) divides the line segment AB in the ratio 3: 4, where the coordinates of A are (2, 5), find the coordiantes of B.



$$\frac{(3 \times x + 4 \times 2)}{3+4} = -1 \quad \Rightarrow \quad \frac{3x+8}{7} = -1$$

$3x + 8 = -7 \quad \Rightarrow \quad 3x = -15$

$x = -5$

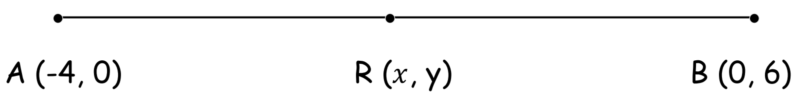
$\therefore$ Coordinates of B are (-5, -2)

$$\frac{(3 \times y + 4 \times 5)}{3+4} = 2 \quad \Rightarrow \quad \frac{3y+20}{7} = 2$$

$3y + 20 = 14 \quad \Rightarrow \quad 3y = 14 - 20$

$3y = -6 \quad \Rightarrow \quad y = -2$

10. The point R divides the line segment AB where A (-4, 0), B (0,6) are such that AR = $\frac{3}{4}$ AB . Find the coordinates of R.



Let coordinates of R be (x, y)

AR $= \frac{3}{4}$ AB (Given)

But, AR + RB $\Rightarrow$ AB $= \frac{3}{4}$ AB + RB = AB

$\Rightarrow$ RB $= AB - \frac{3}{4}AB = \frac{4AB - 3AB}{4} = \frac{AB}{4}$

$$\frac{AR}{RB} = \frac{\frac{3}{4}AB}{\frac{1}{4}AB} = \frac{3}{4} : \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = 3:1$$

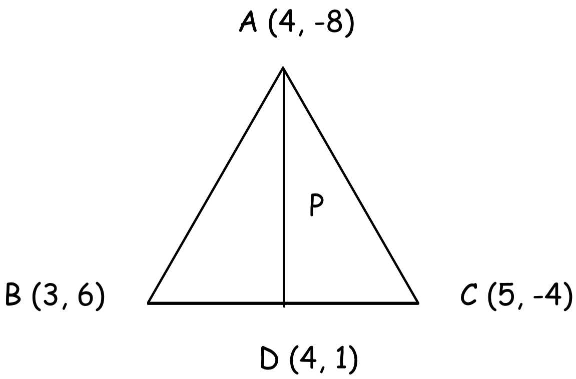
$$x = \frac{3 \times 0 + 1 \times (-4)}{3+1} = \frac{0-4}{4} = \frac{-4}{4} = -1$$

and $y = \frac{3 \times 6 + 1 \times 0}{3+1} = \frac{18+0}{4} = \frac{18}{4} = \frac{9}{2}$

Thus coordinates of R are $\left(-1, \frac{9}{2}\right)$

11. If A (4, -8), B (3, 6) and C (5, -4) are the vertices of Δ ABC, D is the mid point of BC and P is a point on AD joined such that $\frac{AP}{PD}$ = 2, find the coordinates of P.

A (4, -8) B (3,6) and C (5, -4) are verticies of Δ ABC and D is the mid-point of BC



$\therefore$ Coordinates of D are

$\left(\frac{3+5}{2}, \frac{6+(-4)}{2}\right) = \left(\frac{8}{2}, \frac{2}{2}\right)$ = (4, 1)z

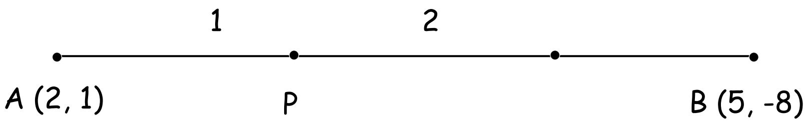
$\frac{AP}{PD} = 2$

$\Rightarrow$ AP : PD = 2 : 1

$\Rightarrow$ Coordinates of P are $\left(\frac{2\times4+1\times3}{2+1}, \frac{2\times1+1\times6}{2+1}\right)$

$= \left(\frac{8+3}{3}, \frac{2+6}{3}\right) = \left(\frac{11}{3}, \frac{8}{3}\right)$

12. The line segment joining the points A (2, 1) and B (5, -8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x$ – y + k = 0. find the value of k



Since point P trisects AB, then PA : PB = 1: 2

Coordinates of P are

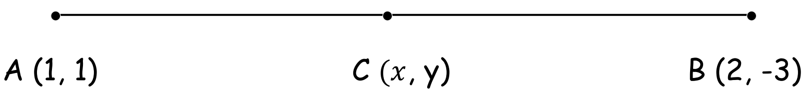
$x = \frac{5+4}{3}$ = 3 and y = $\frac{-8+2}{3}$ = -2

Now P lies on $2x$ – y + k = 0

On putting values of x and y, we get

6 + 2 + k = 0 $\Rightarrow$ k = -8

13. If C is a point lying on the line segment AB joining $A(1, 1)$ and $B(2, -3)$ such that $3AC = CB$, then find the coordinates of C.



$\frac{AC}{CB} = \frac{1}{3}$ (Given)

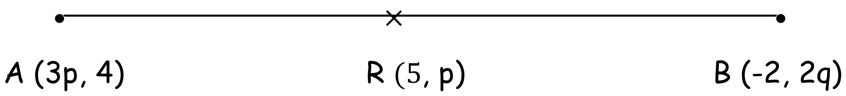
Coordinates of C

$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$\therefore\ x = \frac{2+3}{4} = \frac{5}{4}$ and $y = \frac{-3+3}{1+3} = 0$

$(x, y) = \left(\frac{5}{4}, \mathbf{0}\right)$

14. The coordinates of the mid-point of the line joining the points (3p, 4) and (-2, 2q) are (5, p). Find the values of p and q.



R (5, p) is the mid-point of the line segment joining the points A (3p, 4) and B (-2, 2q).

$\therefore \quad \left(\frac{3p-2}{2}, \frac{4+2q}{2}\right) = (5, p)$

$\Rightarrow \quad \frac{3p-2}{2} = 5$ and $\frac{4+2q}{2} = P$

$\Rightarrow \quad 3p = 10 + 2$ and $4 + 2q = 2p$ ....(ii)

$\Rightarrow \quad 3p = 12 \Rightarrow \quad p = 4$ ....(i)

Substituting p = 4 from (i) in (ii), we get

$4 + 2q = 8 \quad \Rightarrow \quad 2q = 4 \quad \Rightarrow \quad q = 2$

$\therefore \quad p = 4$ and $q = 2$

15. Find the ratio in which the line segment joining (2, -3) and (5, 6) is divided by x – axis,

Let the required ratio be k : 1

Then the coordinates of the point of division are

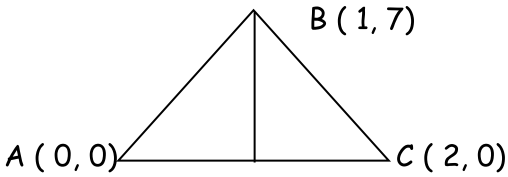
$$\left(\frac{2k+5}{k+1}, \frac{-3k+6}{k+1}\right)$$

This point lies on the x - axis whose equation is y = 0

∴ $\frac{-3k+6}{k+1} = 0 \Rightarrow 3k = 6$, or k = 2

∴ Line segment joining the two points is divided in the ratio 2: 1 internally by x – axis.

16. In given figure BD bisects $\angle B$. Find the length of BD



Here, BD bisects $\angle B$

∴ $\frac{AD}{CD} = \frac{AB}{BC}$

(using angle bisector property) .....(i)

AB = $\sqrt{1^2 + 7^2}$ + = $\sqrt{50}$

BC = $\sqrt{(2-1)^2 + (0-7)^2}$ + = $\sqrt{50}$

∴ (i) becomes

$\frac{AD}{CD} = \frac{\sqrt{50}}{\sqrt{50}} = \frac{1}{1}$ = D bisects AC

Coordinates of D are

$\left(\frac{0+2}{2}, \frac{0+0}{2}\right)$ = (1, 0)

BD = $\sqrt{(1-1)^2 + (0-7)^2}$ = 7

∴ BD = 7 units

17. Find the value of x for which the distance between the point P (2,-3) and Q (x, 5) is 10 unit

According as given PQ = 10 units

$\Rightarrow \qquad \sqrt{(2-x)^2 + (-3-5)^2} = 10$

Squaring both sides, we have

$4 + x^2 - 4x + 64 = 100$

$\Rightarrow \qquad x^2 - 4x - 32 = 0$

$\Rightarrow \qquad x^2 - 8x + 4x - 32 = 0$

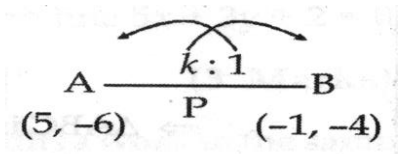
$\Rightarrow \qquad x(x-8) + 4(x-8) = 0$

$\Rightarrow \qquad (x-8)(x+4) = 0$

Δ ABC x = 8 or x = -4

18. Find the ratio in which y - axis divides the line segment joining the points A(5,-6) and B(-1,-4) Also find the coordinates of the point of division.

Let P be a point on the y - axis dividing the line segment AB in the ration k : l using the section formula we get. DIAGRAM



$(0, \alpha) = \left[\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right]$

$\Longrightarrow \qquad \frac{-k+5}{k+1} = 0, \quad \frac{-4k-6}{k+1} = \alpha$

Now, $\frac{-k+5}{k+1} = 0,$

$\Longrightarrow$ -k + 5 = 0

$\Longrightarrow \qquad$ k = 5

$Also \; \frac{-4k-6}{k+1} = \propto$

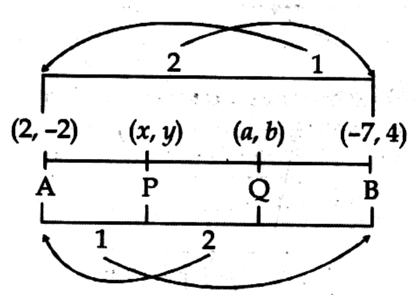
$\Longrightarrow \qquad \frac{-4 \times 5 - 6}{5+1} = \propto$

$\Longrightarrow \qquad \propto = -\frac{13}{3}$

Thus the y-axis divides the line segment in the ration 5 :1

Also the coordinates of the point of division are $\left(0, -\frac{13}{3}\right)$

19. Let P and Q be the points of trisection of the line segment joining the points A (2, -2) and B (-7, 4) such that P is nearer to A. Find the coordinates of P and Q



Using section formula:

P(x, y) $= \left[\left\{\frac{(-7\times1)+(2\times2)}{1+2}\right\}, \left\{\frac{(1\times4)+(2\times(-2))}{1+2}\right\}\right]$

= (-1, 0)

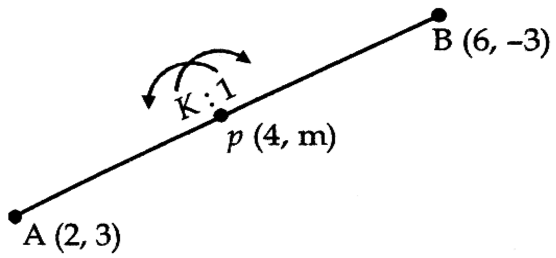
Q(a, b) $= \left[\left\{\frac{(-7\times2)+(1\times2)}{2+1}\right\}, \left\{\frac{(2\times4)+(1\times(-2))}{2+1}\right\}\right]$

$= \left(\frac{-12}{3}, \frac{6}{3}\right)$ = (-4, 2)

20. Find the ration in which P (4, m) divides the line segment joining the points A (2, 3) and B (6,-3). Hence find m,

Let P divides AB in the ration k : 1 By section formula

P → (4, m) → $\left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$



⟹ $\frac{6k+2}{k+1} = 4$

⟹ 6k + 2 = 4k + 4

⟹ 2k = 2

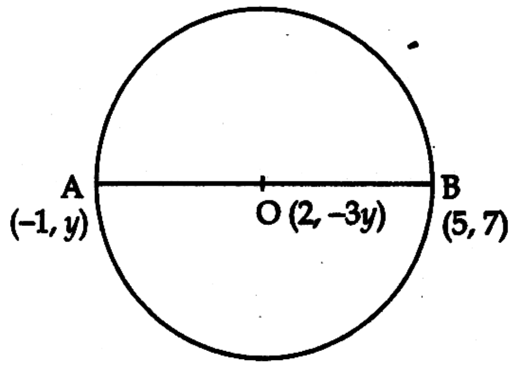
⟹ k = 1

∴ Ratio is 1 : 1

Hence m = $\frac{-3k+3}{k+1} = \frac{-3(1)+3}{1+1} = 0$

$\Rightarrow$ m = 0

21. Point A(-1, y) and B(5, 7) lie on a circle with centre O(2,3y). Find the values of y. Hence find the radius of the circle.



By mid-point formula

$\Rightarrow$ $\frac{y+7}{2} = -3y$

y = -1

Now, A(-1, 1) and O(2, 3)

$\therefore$ Radius = AO = $\sqrt{(2+1)^2 + (3+1)^2}$ = 5 units

22. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q (2,-5) and R (-3,6), find the coordinates of P.

Sol. Let point P be (2a,a)

PQ = PR (given)

$\Rightarrow \sqrt{(2a-2)^2 + \left(a-(-5)\right)^2}$

= $\sqrt{\left(2a-(-3)\right)^2 + (a-6))^2}$

$\Rightarrow \sqrt{(2a-2)^2 + (a \mp 5)^2}$

= $\sqrt{(2a \mp 3)^2 + (a+6)^2}$

$\Rightarrow \sqrt{4a^2 + 4 - 8a + a^2 + 25 + 10a}$

= $\sqrt{4a^2 + 9 + 12a + a^2 + 36 + 10a - 12a}$

$\Rightarrow \sqrt{5a^2 + 2a + 29}$ = $\sqrt{5a^2 + 45}$

Squaring both sides, we get

$5a^2 + 2a + 29 = 5a^2 + 45$

$\Rightarrow \quad 5a^2 + 2a - 5a^2 = 45 - 29$

$\Rightarrow \quad 2a = 16$

$\Rightarrow \quad a = 8$

Thus, the coordinates of the point P are (16, 8)

23. Prove that the points (3, 0), (6, 4), and (-1, 3) are the vertices of a right angled isosceles triangle,

Let A, B and C be the points (3, 0), (6, 4), and (-1, 3) respectively.

Using Distance formula:

AB $= \sqrt{6 - 3^2 + (4 - 0)}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$

BC $= \sqrt{(6 + 1)^2 + (4 - 3)^2}$

$= \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$

ObserveAC^2 = $\sqrt{(3 + 1)^2 + (0 + 3)^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$

AB = AC = 5

$\Rightarrow \Delta ABC$ is isosceles triangle.

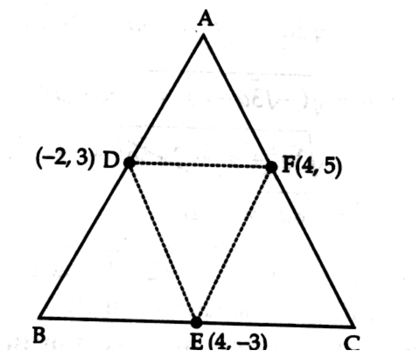
Also, $(5)^2 + (5)^2 = 50 = \left(5\sqrt{2}\right)^2$

$\Rightarrow \quad AC^2 + AB^2 = BC^2$

$\Rightarrow \Delta ABC$ is right angled at A.

24. If (-2,3), (4, -3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of the centroid,

Let the given triangle be ABC and D, E, F are mid-points of sides AB, BC, CA respectively, where D (-2,3), E(4,-3), F(4,5). Since centroid of a triangle is same as centroid of triangle obtained by joining mid-points of sides of main triangle.



∵ Centroid of ΔABC = Centroid of ΔDEF

= $\left(\frac{-2+4+4}{3}, \frac{3-3+5}{3}\right) = \left(2, \frac{5}{3}\right)$.

25. If $\left(1, \frac{p}{3}\right)$ is the mid point of the line segment joining the points (2, 0) and $\left(0, \frac{2}{9}\right)$, then show that the line $5x + 3y + 2 = 0$ passes through the point (-1, 3p).

Using Mid point formula

$$\left(1, \frac{p}{3}\right) = \left(\frac{2+0}{2}, \frac{0+\frac{2}{9}}{2}\right)$$

$$\left(1, \frac{p}{3}\right) = \left(1, \frac{1}{9}\right)$$

$$\Rightarrow \quad \frac{p}{3} = \frac{1}{9}$$

$$\Rightarrow \quad p = \frac{1}{3}$$

∵ (-1, 3p) = $\left(-1, 3 \times \frac{1}{3}\right)$ = (-1, 1)

Plug in x = -1, y = 1 in $5x + 3y + 2 = 0$

5(-1) + 3(1) + 2 = 0

-5 + 5 = 0 (satisfied)

⟹ (-1, 3p) i.e.(-1, 1) lies on line $5x + 3y + 2 = 0$

Short answer type questions II

1. Show that the points A(1, 2), B (5, 4) C (3, 8) and D (-1, 6) are the vertices of a square.

AB = $\sqrt{4^2 + 2^2}$ = $\sqrt{16 + 4}$ = $\sqrt{20}$

BC = $\sqrt{(-2)^2 + (4)^2}$ = $\sqrt{4 + 16}$ = $\sqrt{20}$

CD = $\sqrt{(-4)^2 + (-2)^2}$ = $\sqrt{16 + 4}$ = $\sqrt{20}$

DA = $\sqrt{(-2)^2 + (4)^2}$ = $\sqrt{4 + 16}$ = $\sqrt{20}$

Here AB = BC = CA = DA

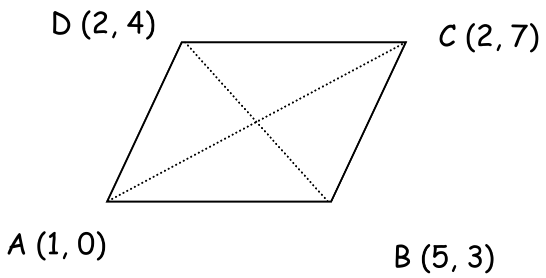
AC = $\sqrt{2^2 + 6^2}$ = $\sqrt{40}$

And BD = $\sqrt{(-6)^2 + (2)^2}$ = $\sqrt{36 + 4}$ = $\sqrt{40}$

All slides of quadrilateral are equal and diagonals are equal.

∴ ABCD is square

2. Show that the points A(1,0), B (5,3) , C (2,7) and D (-2, 4) are the vertices of a parallelogram.



AB = $\sqrt{(5-1)^2 + (3-0)^2}$ = $\sqrt{16 + 9}$ = 5

DC = $\sqrt{(2+2)^2 + (7-4)^2}$ = $\sqrt{16 + 9}$ = 5

BC = $\sqrt{(5-2)^2 + (3-7)^2}$ = $\sqrt{9 + 16}$ = 5

AD = $\sqrt{(1+2)^2 + (0-0)^2}$ = $\sqrt{9 + 16}$ = 5

Or

Alternatively

The given points are the verticies of a parallelogram

Mid point of AC = $\left(\frac{\mathbf{1+2}}{\mathbf{2}}, \frac{\mathbf{0+7}}{\mathbf{2}}\right)$ = $\left(\frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{7}}{\mathbf{2}}\right)$

Mid point of BD = $\left(\frac{\mathbf{5-2}}{\mathbf{2}}, \frac{\mathbf{3+4}}{\mathbf{2}}\right)$ = $\left(\frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{7}}{\mathbf{2}}\right)$

Mid point of AC = mid points of BD

⇒ diagonals bisect each other

∴ The given points are the verticies of a parallelogram

3. Show that points A (7, 5), B (2, 3) and C (6, -7) are the verticies of a right triangle. Also find its area.

AB = $\sqrt{(2-7)^2+(3-5)^2}$ = $\sqrt{25+4}$ = $\sqrt{29}$

BC = $\sqrt{(6-2)^2+(-7-3)^2}$ = $\sqrt{16+100}$ = $\sqrt{116}$

CA = $\sqrt{(7-6)^2+(5+7)^2}$ = $\sqrt{1+144}$ = $\sqrt{145}$

Since, AB^2 + BC^2 = 29 + 116 = 145 = CA^2

∴ Δ ABC is right angles at B

Area = $\frac{1}{2}$ AB x BC

= $\frac{1}{2}$ $\sqrt{29}.\sqrt{116}$ = $\frac{1}{2}$ $\sqrt{29}$ .2. $\sqrt{29}$ = 29

4. Two points A(1, 0) and B (-1, 0) with a variable point P(x, y) satisfy the relation AP – BP = 1. Show that 12x^2- 4y^3 = 3.

Given AP – BP = 1

⇒ $\sqrt{(x-1)^2+(y-0)^2}$ – $\sqrt{(x+1)^2+(y-0)^2}$ = 1

⇒ $\sqrt{(x-1)^2+y^2}$ = 1 + $\sqrt{(x+1)^2+y^2}$

Squaring both sides

$(x-1)^2+y^2$ = 1 $(x+1)^2+y^2$ + 2 $\sqrt{(x-1)^2+y^2}$

⇒ x^2 + 1–2$x = 1 +$ x^2 + 2x + 2 $\sqrt{(x+1)^2+y^2}$

⇒ -1 - 4x = 2 $\sqrt{(x+1)^2+y^2}$

Squaring both sides

⇒ x^2 + 1 - 2x = 1 + x^2 +1 + 2x + 2$\sqrt{(x+1)^2+y^2}$

$\Rightarrow \quad -1 - 4x = 2\sqrt{(x+1)^2 + y^2}$

Squaring both sides

$$(-1-4x)^2 = 4\,(x+1)^2 + y^2$$

$\Rightarrow \quad 1 + 16x^2 + 8x = 4x^2 = 4\ \ x^2 + 1 + 2x + y^2$

$\Rightarrow \quad 12x^2 - 4y^2 = 3$

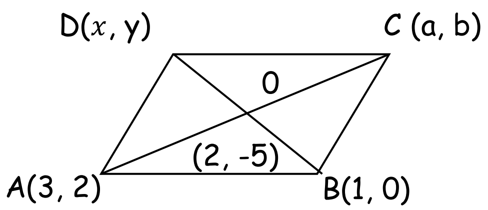
5. If coordinates of two adjacent vertices of a parallelogram are (3, 2), (1, 0) and diagonals bisect each other at (2, -5) find coordinates of the other two vertices.

Let ABCD be a parallelogram, diagonals AC and BD intersects at O.

Let A (3, 2) , B (1, 0) and 0 (2, -5) are coordinates.

Let coordinates of C are (a, b) and coordinates of D are (x, y). As diagonals of parallelogram bisect each other at 0. So, O is mid point of AC and BD.

$\therefore \quad 2 = \frac{3+a}{2}$ and $-5 = \frac{2+b}{2}$



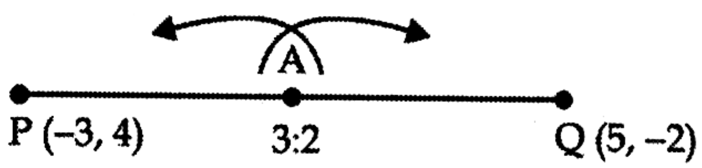
$\Rightarrow \quad a = 1$ and $b = -12$

Also, $\quad \frac{1+x}{2} = 2$ and $\frac{0+y}{2} = -5$

$x = 3$ and $y = -10$

$\therefore$ Coordinates are (1, -12) and (3, -10)

6. Given two fixed two point P (-3, 4) and Q (5, -2). Find the coordinates of points A and B in PQ such that 5 PA = 3 PQ and 3PB = 2 PQ.



Here 5PA = 3 PQ

$\Rightarrow \quad \frac{PQ}{PA} = \frac{5}{3}$

$\Rightarrow \quad \frac{PA + AQ}{PA} = \frac{5}{3}$

$\Rightarrow \quad \frac{PA}{PA} + \frac{AQ}{PA} = \frac{5}{3}$

$\Rightarrow \quad 1 + \frac{AQ}{PA} = \frac{5}{3}$

$\Rightarrow \quad \frac{AQ}{PA} = \frac{5}{3} - 1 \quad \Rightarrow \frac{AQ}{PA} = \frac{2}{3}$ or $\frac{PA}{AQ} = \frac{3}{2}$

Thus A divides PQ in the ratio 3: 2 internally. Using section formula

A $\left[\frac{3(5)+2(-3)}{3+2}, \frac{3(-2)+2(4)}{3+2}\right] = \left(\frac{9}{5}, \frac{2}{5}\right)$

Again 3PB = 2 PQ

$\Rightarrow \quad \frac{PQ}{PB} = \frac{3}{2}$

$\Rightarrow \quad \frac{PB+ BQ}{PB} = \frac{3}{2}$

$\Rightarrow \quad \frac{PB+ BQ}{PB} = \frac{3}{2}$

$\Rightarrow \quad \frac{PB}{PB} + \frac{BQ}{PB} = \frac{3}{2} \Rightarrow 1 + \frac{BQ}{PB} = \frac{3}{2}$

$\Rightarrow \quad \frac{BQ}{PB} = \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow \quad \frac{PB}{BQ} = \frac{2}{1}$

Thus B divides PQ in the ratio 2: 1. Using section formula.

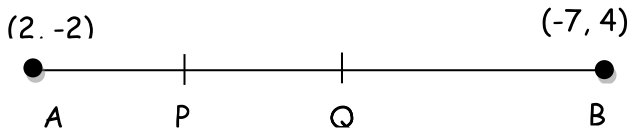
B is $\left[\frac{2(5)+1(-3)}{2+1}, \frac{2(-2)+1(4)}{2+1}\right] = \left(\frac{7}{3}, 0\right)$

Thus A is $\left(\frac{9}{5}, \frac{2}{5}\right)$ and B is $\left(\frac{7}{3}, 0\right)$

7. Find the coordinates of the points of trisection of the line segment joining the points A (2, -2) and B (-7, 4)

Let P and Q be the points of trisection of AB

∴ P divides AB in the ratio 1: 2 and Q divides AB in the ratio 2: 1 internally.



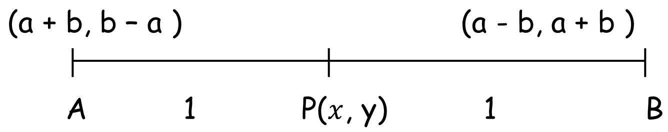
Applying section formula, coordinates of P are

P $\rightarrow \left[\frac{\mathbf{1(-7)+2\,(2)}}{\mathbf{1+2}}, \frac{\mathbf{1(4)+1\,(-2)}}{\mathbf{1+2}}\right]$ i.e. P (-1, 0)

Similarly, coordinates of Q are

$\left[\frac{\mathbf{2(-7)+1\,(2)}}{\mathbf{2+1}}, \frac{\mathbf{2(4)+1\,(-2)}}{\mathbf{2+1}}\right]$ i.e. Q (-4, 2)

8. If the point P (x, y) is equidistant from the points A (a + b, b - a) and B (a - b, a + b). Prove that bx = ay.



Using mid-point formula

$(x, \text{y}) = \left[\frac{\mathbf{a + b + a - b}}{\mathbf{2}}, \frac{\mathbf{b - a + a + b}}{\mathbf{2}}\right]$

$\Rightarrow \quad x = \frac{\mathbf{a+b+a-b}}{\mathbf{2}} = \text{a}$ ......(1)

$\Rightarrow \quad \text{y} = \frac{\mathbf{b-a+a+b}}{\mathbf{2}} = \text{b}$ ......(2)

(1) + (2) gives = $\frac{\boldsymbol{x}}{\boldsymbol{y}} = \frac{\boldsymbol{a}}{\boldsymbol{b}}$

$\Rightarrow$ bx = ay (Proved)

9. The vertices of a triangle are (-2, 0), (2, 3) and (1, -3). Is the triangle equilateral, isosceles or scalene?

Let the given points be A (-2, 0), B (2, 3) and (1, -3) .

$\therefore$ AB = $\sqrt{(-2-2)^2(0-3)^2}$

= $\sqrt{16+9}$ = $\sqrt{25}$ = 5

BC = $\sqrt{(2-1)^2(3+3)^2}$

= $\sqrt{1+36}$ = $\sqrt{37}$

CA = $\sqrt{(1+2)^2(-3-0)^2}$

= $\sqrt{9+9}$ = $\sqrt{18}$ = $3\sqrt{2}$

Clearly 5 $\neq$ 37 = $3\sqrt{2}$

i.e. AB $\neq$ BC $\neq$ CA

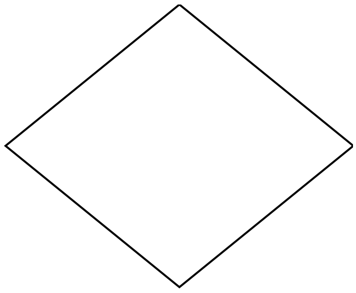
$\Rightarrow \Delta$ ABC is a scalene triangle

10. Find the area of a rhombus if its vertices are (3, 0) (4, 5), (-1, 4) and (-2, -1) taken in order

Let ABCD be given rhombus with A (3, 0), B (4, 5), (-1, 4) and (-2, -1)

D (-2, -1)

A(3, 0)



C (-1, 4)

B (4, 5)

Thus AC and BD are its diagonals

Using distance formula

AC = $\sqrt{(3+1)^2(0-4)^2}$

$\sqrt{16+16}$ = $\sqrt{32}$ = 4 $\sqrt{2}$ units

BD = $\sqrt{(4+2)^2(5+1)^2}$

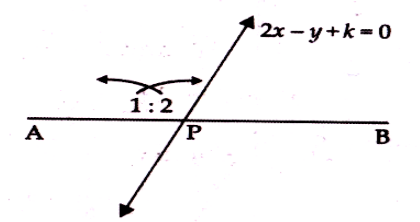
$\sqrt{36+36}$ = 6 $\sqrt{2}$ units

Since area of rhombus = $\frac{1}{2}$ (Product of the diagonals)

= $\frac{1}{2}$ (4 $\sqrt{2}$) 6 $\sqrt{2}$ = 24 sq. units

11. Find the point on y –axis which is equidistant from the point (5, -2) and (-3, 2)

Let the required point on y- axis be P (0, b) and given points be A (5, 2) and B (-3, 2)



According to question

PA = PB

⇒ $PA^2 = PB^2$

⇒ $(5-0)^2 + (-2-b)^2 = (-3-0)^2 + (-2-b)^2$

⇒ 29 + 4b + b^2 = 13 + b^2- 4b

⇒ b = -2

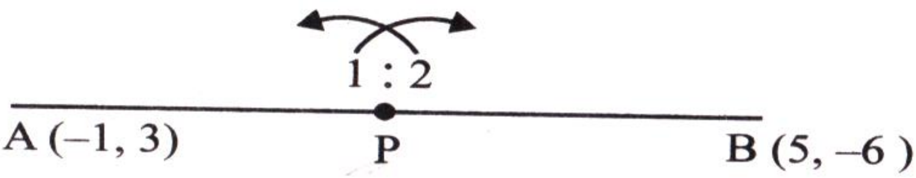
∴ Required point is (0, -2)

12. The line segment joining the points A (2, 1) and B (5, -8) is trisected by the points P and Q, where P is nearer to A. If the point P also lies on the line 2x – y + k = 0, find the value of k.

Or

Show that (a, a), (-a, -a) and (-$\sqrt{3}a$, $\sqrt{3}a$) are verticies of an equilateral triangle.

Since, P lies nearer to A and is one of the point of trisection.



∴ P divides AB in the ratio 1 : 2 So, by section formula

P $\left[\frac{1\times5+2\times2}{1+2}, \frac{1\times(-8)+2\times1}{1+2}\right]$

= P$\left(\frac{9}{3}, \frac{-6}{3}\right)$ = P (3, -2)

Since P lies on 2x – y + k =0

∴ Coordinates of P must satisfy

⇒ 2(3) – (-2) + k = 0

⇒ 10 + k = 0

⇒ k = -10

Or

Let given point be A (a, a) B (-a, -a) and C (-$\sqrt{3}a$, $\sqrt{3}a$)

Now using Distance Formula

AB = $\sqrt{(a(-a)^2+(a(-a)^2}$

= $\sqrt{(2a)^2+(2a)^2}$ =$\sqrt{8a^2} = 2\sqrt{2}a$

BC = $\sqrt{(-a+\sqrt{3}\,a)^2+(-a-\sqrt{3}\,a)^2}$

= $\sqrt{a^2+3a^2-\cancel{2\sqrt{3}a^2}+a^2+3a^2+\cancel{2\sqrt{3}a^2}}$

= $\sqrt{(2a)^2+(6a)^2}$ =$\sqrt{8a^2} = 2\sqrt{2}a$

CA = $\sqrt{(-\sqrt{3}\,a-a)^2+(\sqrt{3}\,a-a)^2}$

= $\sqrt{3a^2+a^2+2\sqrt{3}\,a^2+3a^2+a^2-2\sqrt{3}\,a^2}$

= $\sqrt{8a^2} = 2\sqrt{2}a$

Since AB = BC =CA = ($2\sqrt{2}a$ each)

∴ Δ ABC is an equilateral triangle

13. Show that Δ ABC, where A (-2, 0), B (2, 0), C (0, 2) and Δ PQR where P (-4, 0), Q (4, 0) R (0, 4) are similar triangles

Using distance formula

AB = $\sqrt{-2-2)^2+(0-0)^2}$ = 4 units

BC = $\sqrt{(2-0)^2+(0-2)^2}$

= $\sqrt{4+4}$ = $2\sqrt{2}$ units

AC = $\sqrt{-2-0)^2+(0-2)^2}$

= $\sqrt{4+4}$ = $2\sqrt{2}$ units

PQ =$\sqrt{(-4-4)^2(0-0)^2}$ = $\sqrt{64}$ = 8 units

QR = $\sqrt{(4-0)^2+(0-4)^2}$ = $\sqrt{16+16}$

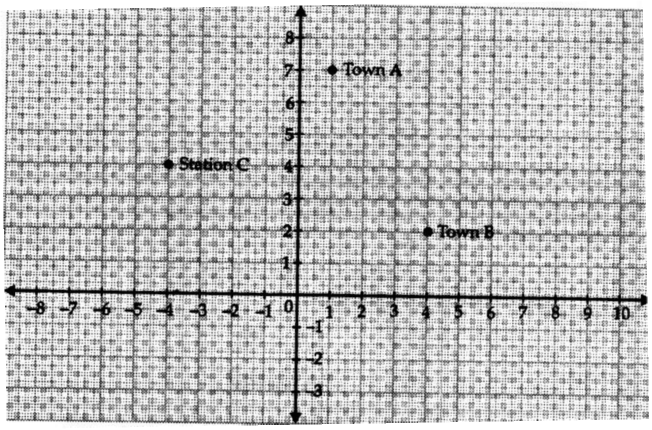
= $\sqrt{32}$ = $4\sqrt{2}$ units

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{2}$

i.e. sides of Δs are proportional

⇒ Δ ABC ~ Δ PQR

14. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation, answer the following questions.



i. **Who will travel more distance, Seema or Aditya, to reach to their hometown?**

ii. **Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the point5s represented by Town A and Town B. Find the coordinates of the point represented by the point D.**

iii. **Find the area of the triangle formed by joining the points represented by A,B and C.**

Sol. i. Reading the coordinates of A,B,C as given below A(1, 7), B(4, 2),C(-4, 4)

Distance travelled by Seema = AB

= $\sqrt{(1-4)^2 + (7-2)^2}$ = $\sqrt{9+25}$ = $\sqrt{34}$

Distance travelled by Aditya

= BC = $\sqrt{(4+4)^2 + (2-4)^2}$ = $\sqrt{64+4}$ = $\sqrt{68}$

Ce $\sqrt{68} > \sqrt{34}$

∴ Aditya travels more distance.

ii. Using mid-point formula, The coordinates of D are

= $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

iii. Area of ΔABC = $\frac{1}{2}|\,1(2-4)+4(4-7)-4(7-2)|$

= $\frac{1}{2}$ | -2 - 12 - 20 | = $\frac{1}{2}$ | - 34 |

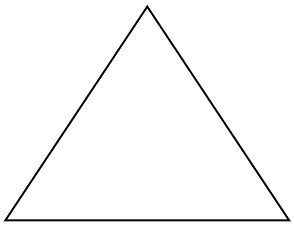
= 17 sq. Units

Long answer type questions

1. One vertex of an equilateral triangle with side 2 units is at origin. Another vertex lies on the line $x = \sqrt{3y}$ and is in first quadrant. Find the coordinates of other two vertices.

Here , one vertex O is at origin, other vertex A (h, k) is on the line $x = \sqrt{3y}$ and third vertex is B $(\alpha,\ \beta)$

B (α, β)



O (0, 0) A (h, k)

∴ ΔOAB is an equilateral Δ OA = 2 units

⇒ $\sqrt{h^2+k^2} = 2$ ⇒ $h^2+k^2 = 4$ .....(i)

Also, A (h, k) is on the line $x = \sqrt{3y}$

⇒ h = $\sqrt{3}$k .....(ii)

Substituting in (i), we get

$(\sqrt{3}k)^2 + k^2 = 4$

⇒ $4k^2 = 4$ ⇒ $k^2 = 1$

⇒ k = ± 1

But A is in 1st quadrant ∴ k = ≠ 1 when k = 1 , eq (ii) becomes

$$h = \sqrt{3} \times 1 = \sqrt{3}$$

∴ Coordinated of A are $(\sqrt{3}\ , 1)$

Now OB = 2

$\Rightarrow \quad \sqrt{\alpha^2 + \beta^2} = 2$

$\Rightarrow \quad \sqrt{\alpha^2 + \beta^2} = 4$

Also AB = 2

$\Rightarrow \quad \sqrt{(\alpha - b)^2 + (\beta - k)^2} = 4$

$\Rightarrow \quad (\alpha - \sqrt{3})^2 + (\beta - 1k)^2 = 4$

$\Rightarrow \quad \alpha^2 + 3 - 2\sqrt{3}\,\alpha + \beta^2 + 1 - 2\beta = 4$

$\Rightarrow \quad \alpha^2 + \beta^2 - 2\sqrt{3}\,\alpha - 2\beta = 0$

$\Rightarrow \quad 4 - 2\sqrt{3}\,\alpha - 2\beta = 0$ [Using (iii)]

$$\sqrt{3}\,\alpha + \beta = 2$$

$\Rightarrow \quad \beta = 2 - \sqrt{3}\,\alpha$ ...(iv)

Substituting in (iii), we get

$\alpha^2 + (2 - \sqrt{3}\,\alpha)^2 = 4 \Rightarrow \alpha^2 + 4 + 3\,\alpha^2 - 4\sqrt{3}\,\alpha = 4$

$\Rightarrow 4\alpha^2 - 4\sqrt{3}\,\alpha = 4 \Rightarrow 4\alpha\,(\alpha - \sqrt{3}\,) = 0$

$\Rightarrow \alpha = 0$ or $\alpha = \sqrt{3}$

When $\alpha = 0$ eq.(iv) becomes $= \beta = 2 - \sqrt{3} \times 0 = 2$

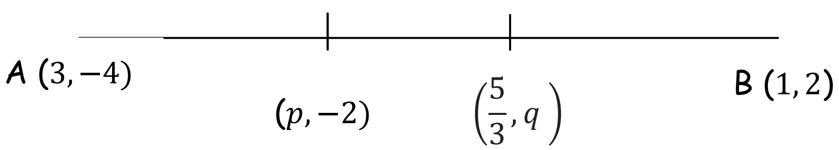
Coordinates of B are (0, 2)

When $\alpha = \sqrt{3}$ eq.(iv) becomes $= \beta = 2 - \sqrt{3} \times \sqrt{3} = -1$

Coordinates of B are $(\sqrt{3}, -1)$

2. The line segment AB joining the points A (3, -4) and B (1, 2) is trisected at the points P (p, -2) and Q $\left(\frac{5}{3}, q\right)$. Find the values of p and q

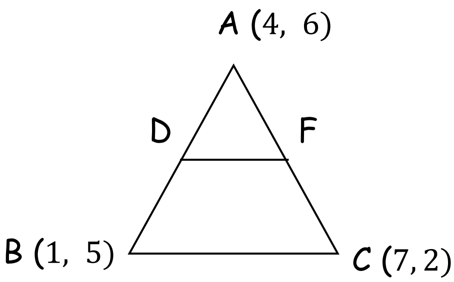
Now AP : PB = 1: 2



∴ $p = \frac{1 \times 1 + 2 \times 3}{1+2} \Rightarrow p = \frac{7}{3}$

Also AQ : QB = 2: 1 $\Rightarrow q = \frac{2 \times 2 + 1 \times 4}{1 + 2}$ = 0

3. In figure, the vertices of Δ ABC are A (4, 6), B (1, 5) and C (7, 2) . A line- segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of Δ ADE and compare it with area of Δ ABC.



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{AD}{AB-AD} = \frac{AE}{AC-CE} = \frac{1}{3-1}$$

$$\Rightarrow \quad \frac{AD}{BD} = \frac{AE}{EC} = \frac{1}{2}$$

⇒ AD : BD = 1 : 2 AE : EC = 1 : 2

Using section formula

$D = \left\{\frac{2 \times 4 + 1 \times 1}{3}, \frac{2 \times 6 + 1 \times 5}{3}\right\} = \left[3, \frac{17}{3}\right]$

$E = \left\{\frac{2 \times 4 + 1 \times 7}{3}, \frac{2 \times 6 + 1 \times 2}{3}\right\} = \left[5, \frac{14}{3}\right]$

Area of Δ ADB

$= \frac{1}{2} + \left|4\left[\frac{17}{3} - \frac{14}{3}\right] - 3\left[6 - \frac{14}{3}\right] + 5\left[6 - \frac{17}{3}\right]\right|$

= $\frac{1}{2}$ + $\left|\left[4+(-12)+\frac{5}{3}\right]\right|$ = $\frac{1}{2}$ $\left|\left[\frac{12-36+5}{3}\right]\right|$

= $\frac{1}{2}$ $\left|\left[-\frac{19}{3}\right]\right|$ = $\frac{1}{2}$ × $\frac{19}{3}$

= $\frac{19}{6}$ Sq. units = 3.16 Sq. units

Area of Δ ABC

$\frac{1}{2}$ $|[4\,(5-2)+1\,(2-6)+7(6-5)]|$

$\frac{1}{2}$ $|[12-4+7]|$ = $\frac{15}{2}$ = 7.5 Sq. units

So, area of Δ ABC is more than that of Δ ADB by 4. 34 Sq. units.

4. Determine the ratio in which the line 3x + y - 9 = 0 divides the segment joining the points (1,3) and (2,7).

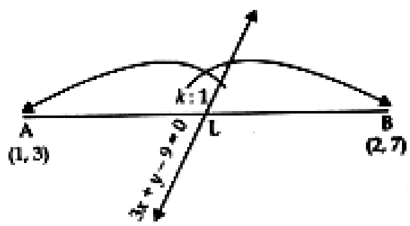
Let line 3x + y – 9 = 0 divides the segment joining the points A (1, 3) and B (2, 7) in the ratio k : 1 at point L.

Using section formula

Co-ordinates of L are

x = $\frac{k \times 2+1 \times 1}{k+1}$

x = $\frac{2k+1}{k+1}$



and y = $\frac{7 \times k+3 \times 1}{k+1}$

y = $\frac{7k+3}{k+1}$

New point L lies on line 3x + y – 9 = 0

i.e. 3 $\left[\frac{2k+1}{k+1}\right]$ + $\left[\frac{7k+3}{k+1}\right]$ - 9 = 0

⇒ $\frac{6k+3+7k+3-9k-9}{k+1}$ = 0

$4k - 3 = 0$

$K = \frac{3}{4}$

Hence required ratio be 3 : 4